

Magnetic dipole moments of the negative parity $J^{PC} = 2^{--}$ mesons in QCD

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Abstract

Magnetic dipole moments of the negative parity light and heavy tensor mesons are calculated within the light cone QCD sum rules method. The results are compared with the positive parity counterparts of the corresponding tensor mesons. The results of the analysis show that the magnetic dipole moments of the negative parity light mesons are smaller compared to those of the positive parity mesons. Contrary to the light meson case, magnetic dipole moments of the negative parity heavy mesons are larger than the ones for the positive parity mesons.

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1 Introduction

The study of the spectroscopy of particles play critical role for understanding the dynamics of quantum chromodynamics (QCD), both at large and short distances. According to the conventional quark model, the particles are characterized by the J^{PC} quantum numbers $P = (-1)^{L+1}$ and $C = (-1)^{L+S}$, where L and S are the orbital angular momentum and the total spin, respectively. The spectroscopy of the particles with the quantum numbers $J^{PC} = 0^{\pm+}, 1^{\pm-}, 1^{++}$ are widely investigated elsewhere in the literature. The mass and residues of light tensor mesons are studied firstly in [1] in the framework of the QCD sum rules method. Later similar studies are extended for the strange tensor mesons in [2]. The masses and decay constants of the ground states of the heavy χ_{Q_2} tensor mesons are investigated within the same framework in [3]. The relevant quantities in understanding the internal structure of the mesons and baryons are their electromagnetic form factors and multipole moments, such as the dipole moments. The dipole moments for the heavy and light tensor mesons with the quantum number $J^P = 2^+$ are investigated in the framework of the light cone QCD sum rules in [4] and [5], respectively. The negative parity tensor mesons have received less attention. The first attempt has recently been made to calculate the mass and decay constants of the negative parity $\bar{q}q$, $\bar{q}s$, $\bar{s}s$, $\bar{q}c$, $\bar{s}c$, $\bar{q}b$, $\bar{s}b$, and $\bar{c}b$ tensor mesons within the QCD sum rules method in [6].

In the present work we calculate the magnetic dipole moments of these negative parity tensor mesons in the framework of the light cone QCD sum rules method (for more about the light cone QCD sum rules method, see [7] and [8]).

The paper is organized as follows. Section 2 is devoted to the derivation of the light cone QCD sum rules for the magnetic dipole moment of the negative parity mesons $\bar{q}q$, $\bar{q}s$, $\bar{s}s$, $\bar{q}c$, $\bar{s}c$, $\bar{q}b$ and $\bar{s}b$. In section 3, numerical analysis of the obtained sum rules for the dipole moments of the 2^{--} tensor mesons is performed. This section also contains the discussions, and brief summary of the present study.

2 Theoretical framework

In this section we derive the light cone sum rules for the magnetic dipole moments of the negative parity tensor mesons. For this goal we consider the 3-point correlation function,

$$\Pi_{\mu\nu\rho\alpha\beta}(p, q) = - \int d^4x \int d^4y e^{i(px+qy)} \left\langle 0 \left| \mathcal{T} \left\{ j_{\mu\nu}(0) j_{\rho}^{el}(y) \bar{j}_{\alpha\beta}(x) \right\} \right| 0 \right\rangle, \quad (1)$$

where $j_{\mu\nu}$ is the tensor interpolating current with $J^{PC} = 2^{--}$,

$$j_{\mu\nu} = \frac{i}{2} \left[\bar{q}_1(x) \gamma_{\mu} \gamma_5 \overleftrightarrow{D}_{\nu} q_2(x) + \bar{q}_2(x) \gamma_{\nu} \gamma_5 \overleftrightarrow{D}_{\mu} q_1(x) \right]. \quad (2)$$

The electromagnetic current j_{ρ}^{el} in Eq. (1) is defined as,

$$j_{\rho}^{el} = e_{q_1} \bar{q}_1 \gamma_{\rho} q_1 + e_{q_2} \bar{q}_2 \gamma_{\rho} q_2,$$

where e_{q_i} is the electric charge of the corresponding quark. The momenta p and q are carried by the currents $j_{\mu\nu}$ and j_{ρ}^{el} , respectively.

The covariant derivative $\overleftrightarrow{\mathcal{D}}$ is defined as

$$\overleftrightarrow{\mathcal{D}}_\mu(x) = \frac{1}{2} \left[\overrightarrow{\mathcal{D}}_\mu(x) - \overleftarrow{\mathcal{D}}_\mu(x) \right], \quad (3)$$

where

$$\begin{aligned} \overrightarrow{\mathcal{D}}_\mu(x) &= \overrightarrow{\partial}_\mu(x) - i\frac{g}{2}\lambda^a A_\mu^a(x), \\ \overleftarrow{\mathcal{D}}_\mu(x) &= \overleftarrow{\partial}_\mu(x) + i\frac{g}{2}\lambda^a A_\mu^a(x). \end{aligned}$$

Here $A_\mu^a(x)$ is the gluon field, satisfying the Fock-Schwinger gauge condition $x^\mu A_\mu^a(x) = 0$, which we have used in the present work, and λ^a are the Gell-Mann matrices.

The correlation function can be rewritten in terms of the external background electromagnetic field. For this goal it is necessary to introduce plane wave electromagnetic field,

$$F_{\mu\nu} = i(q_\mu \varepsilon_\nu - q_\nu \varepsilon_\mu) e^{iqx},$$

where ε_μ is the polarization vector, q_μ is the four-momentum vector of the background electromagnetic field, and the radiated photon can be absorbed into the background field. This allows us to rewrite the correlation function as,

$$\varepsilon^\rho \Pi_{\mu\nu\rho\alpha\beta} = i \int d^4x e^{ipx} \left\langle 0 \left| T \left\{ j_{\mu\nu}(x) \bar{j}_{\alpha\beta}(0) \right\} \right| 0 \right\rangle_F, \quad (4)$$

where the subindex F means that the vacuum expectation value is calculated in the background electromagnetic field.

Note that the correlation function given in Eq. (1) can be obtained from Eq. (4) by expanding it in powers of $F_{\mu\nu}$, and taking only the terms linear in $F_{\mu\nu}$ (more technical details about the external background field method can be found in [9, 10]). The main advantage of using the background field method is that it separates the hard and soft contributions in a gauge invariant way. Hence, the main object in our discussion is the correlation function given in Eq. (4). Here a cautionary note is in order. Since the current $J_{\mu\nu}$ contains derivatives, we first replace $\bar{j}_{\alpha\beta}(0)$ in Eq. (4) with $\bar{j}_{\alpha\beta}(y)$, and after carrying out the calculations we set the variable y to zero.

In order to obtain the sum rules for the dipole magnetic moment the tensor mesons, one should insert the spin-2 mesons into the correlation function, as the result of which we obtain,

$$\Pi_{\mu\nu\alpha\beta} = i \frac{\langle 0 | j_{\mu\nu} | T(p, \epsilon) \rangle \langle T(p, \epsilon) | T(p+q, \epsilon) \rangle_F}{p^2 - m_T^2} \frac{\langle T(p+q, \epsilon) | \bar{j}_{\alpha\beta} | 0 \rangle}{(p+q)^2 - m_T^2} + \dots \quad (5)$$

The matrix element $\langle 0 | j_{\mu\nu} | T(p, \epsilon) \rangle$ is defined as,

$$\langle 0 | j_{\mu\nu} | T(p, \epsilon) \rangle = f_T m_T^3 \epsilon_{\mu\nu}, \quad (6)$$

where f_T is the decay constant of the tensor meson, and $\epsilon_{\mu\nu}$ is the polarization tensor.

In the presence of the background electromagnetic field, the transition matrix element $\langle T(p\epsilon)|T(p+q, \epsilon)\rangle_F$ is parametrized as follows:

$$\begin{aligned} \langle T(p, \epsilon)|T(p+q, \epsilon)\rangle_F = & \epsilon_{\alpha'\beta'}^*(p) \left\{ 2(\epsilon' p) \left[g^{\alpha'\rho} g^{\beta'\sigma} F_1(q^2) - g^{\beta'\sigma} \frac{q^{\alpha'} q^\rho}{2m_T^2} F_3(q^2) + \frac{q^{\alpha'} q^\rho}{2m_T^2} \frac{q^{\beta'} q^\sigma}{2m_T^2} F_5(q^2) \right] \right. \\ & \left. + (\epsilon'^\sigma q^{\beta'} - \epsilon'^{\beta'} q^\sigma) \left[g^{\alpha'\rho} F_2(q^2) - \frac{q^{\alpha'} q^\rho}{2m_T^2} F_4(q^2) \right] \right\} \epsilon_{\rho\sigma}(p+q) , \end{aligned} \quad (7)$$

where $F_i(q^2)$ are the form factors.

In analysis of the experimental data it is more convenient to use the form factors of a definite multipole in a given reference frame. Relations between these two sets of form factors for the arbitrary integer, and half-integer spin are derived in [11], and the relations for the real photon case are:

$$\begin{aligned} F_1(0) &= G_{E_0}(0) , \\ F_2(0) &= G_{M_1}(0) , \\ F_3(0) &= -2G_{E_0}(0) + G_{E_2}(0) + G_{M_1}(0) , \\ F_4(0) &= -G_{M_1}(0) + G_{M_3}(0) , \\ F_5(0) &= G_{E_0}(0) - [G_{E_2}(0) + G_{M_1}(0)] + G_{E_4}(0) + G_{M_3}(0) , \end{aligned} \quad (8)$$

where $G_{E_\ell}(0)$ and $G_{M_\ell}(0)$ are the electric and magnetic multipoles. Substituting these form factors in Eq. (7) we get,

$$\begin{aligned} \langle T(p, \epsilon) | T(p+q, \epsilon') \rangle_F = & \epsilon_{\alpha'\beta'}^*(p) \left\{ 2(\epsilon' \cdot p) \left[g^{\alpha'\lambda} g^{\beta'\sigma} G_{E_0} - \frac{q^{\alpha'} q^\lambda}{2m_T^2} g^{\beta'\sigma} (-2G_{E_0} + G_{E_2} + G_{M_1}) \right. \right. \\ & \left. \left. + \frac{q^{\alpha'} q^\lambda}{2m_T^2} \frac{q^{\beta'} q^\sigma}{2m_T^2} (G_{E_0} - G_{E_2} - G_{M_1} + G_{E_4} + G_{M_3}) \right] \right. \\ & \left. + (\epsilon'^\sigma q^{\beta'} - \epsilon'^{\beta'} q^\sigma) \left[g^{\alpha'\lambda} G_{M_1} - \frac{q^{\alpha'} q^\lambda}{2m_T^2} (-G_{M_1} + G_{M_3}) \right] \right\} \epsilon^{\lambda\sigma}(p+q) \quad (9) \end{aligned}$$

In order to obtain the correlation function from the physical side, we substitute Eqs. (6) and (9) into Eq. (5), and perform summation over the spins of the tensor particles by using,

$$\epsilon_{\mu\nu}(p) \epsilon_{\alpha\beta}^*(p) = \frac{1}{2} \mathcal{P}_{\mu\alpha} \mathcal{P}_{\nu\beta} + \frac{1}{2} \mathcal{P}_{\mu\beta} \mathcal{P}_{\nu\alpha} - \frac{1}{3} \mathcal{P}_{\mu\nu} \mathcal{P}_{\alpha\beta} , \quad (10)$$

where

$$\mathcal{P}_{\mu\nu} = -g_{\mu\nu} + \frac{p_\mu p_\nu}{m_T^2} ,$$

we obtain,

$$\Pi_{\mu\nu\rho\alpha\beta}(p, q) \epsilon^\rho = \frac{m_{T_Q}^6 g_{T_Q}^2}{(p^2 - m_{T_Q}^2)[(p+q)^2 - m_{T_Q}^2]} \left\{ \frac{1}{2} \mathcal{P}_{\mu\alpha'}(p) \mathcal{P}_{\nu\beta'}(p) + \frac{1}{2} \mathcal{P}_{\mu\beta'}(p) \mathcal{P}_{\nu\alpha'}(p) \right.$$

$$\begin{aligned}
& -\frac{1}{3}\mathcal{P}_{\mu\nu}(p)\mathcal{P}_{\alpha'\beta'}(p)\Big\}\times\Big\{2(p\cdot\varepsilon)\Big[g^{\alpha'\lambda}g^{\beta'\sigma}G_{E_0}(0)-g^{\beta'\sigma}\frac{q^{\alpha'}q^\lambda}{2m_{T_Q}^2}\Big(-2G_{E_0}(0)+G_{E_2}(0) \\
& +G_{M_1}(0)\Big)+\frac{q^{\alpha'}q^\lambda}{2m_{T_Q}^2}\frac{q^{\beta'}q^\sigma}{2m_{T_Q}^2}\Big(G_{E_0}(0)-[G_{E_2}(0)+G_{M_2}(0)]+G_{E_4}(0)+G_{M_3}(0)\Big)\Big] \\
& +\Big(\varepsilon^\sigma q^{\beta'}-\varepsilon^{\beta'}q^\sigma\Big)\Big[g^{\alpha'\lambda}G_{M_1}(0)-\frac{q^{\alpha'}q^\lambda}{2m_{T_Q}^2}\Big(-G_{M_1}(0)+G_{M_2}(0)\Big)\Big]\Big\} \\
& \times\Big\{\frac{1}{2}\mathcal{P}_{\lambda\alpha}(p+q)\mathcal{P}_{\sigma\beta}(p+q)+\frac{1}{2}\mathcal{P}_{\lambda\beta}(p+q)\mathcal{P}_{\alpha\sigma}(p+q)-\frac{1}{3}\mathcal{P}_{\lambda\sigma}(p)\mathcal{P}_{\alpha\beta}(p+q)\Big\}. \quad (11)
\end{aligned}$$

One can easily see that the expression of the correlation function contains many independent structures, and any one of these structures can be used in the analysis of the multipole moments of the tensor mesons. In this work we restrict ourselves to calculate the magnetic dipole form factor only and for this aim we choose the structure $(\varepsilon'^\beta q^\nu - \varepsilon'^\nu q^\beta)g^{\mu\alpha}$, whose coefficient is,

$$\Pi = \frac{m_T^6 g_T^2}{(p^2 - m_T^2)[(p+q)^2 - m_T^2]} \left\{ \frac{1}{4}G_{M_1} + \text{other structures} \right\} + \dots \quad (12)$$

The choice of of this structure is dictated by the fact that it does not contain any contribution from the contact terms (see [12]).

Using the operator product expansion (OPE), we calculate the correlation from the QCD side in deep Euclidean region where $p^2 \rightarrow -\infty$ and $(p+q)^2 \rightarrow -\infty$. After contracting all quark fields we obtain,

$$\begin{aligned}
\Pi_{\mu\nu\alpha\beta} = & \frac{-i}{16} \int e^{ip\cdot x} e^{-i(p+q)\cdot y} d^4x d^4y \left\langle 0 \left| \left\{ S_{q_1}(y-x)\gamma_\mu\gamma_5 \left[\vec{\partial}_\nu(x) \vec{\partial}_\beta(y) - \vec{\partial}_\nu(x) \overleftarrow{\partial}_\beta(y) \right. \right. \right. \right. \\
& - \left. \overleftarrow{\partial}_\nu(x) \vec{\partial}_\beta(y) + \overleftarrow{\partial}_\nu(x) \overleftarrow{\partial}_\beta(y) \right] S_{q_2}(x-y)\gamma_\alpha\gamma_5 \Big\} \Big| 0 \Big\rangle_F + \{\beta \leftrightarrow \alpha\} + \{\nu \leftrightarrow \mu\} \\
& + \{\beta \leftrightarrow \alpha, \nu \leftrightarrow \mu\}. \quad (13)
\end{aligned}$$

As has been noted, we set $y = 0$ after performing the derivatives with respect to y . It follows from Eq. (13) that in calculation of the correlation function the quark operators are needed. The expression of the light quark operator is given as,

$$\begin{aligned}
S_q(x-y) = & S^{free}(x-y) - \frac{\langle qq \rangle}{12} \left[1 - i\frac{m_q}{4}(\not{x} - \not{y}) \right] - \frac{(x-y)^2}{192} m_0^2 \langle qq \rangle \left[1 - i\frac{m_q}{6}(\not{x} - \not{y}) \right] \\
& - ig_s \int_0^1 du \left\{ \frac{\not{x} - \not{y}}{16\pi^2(x-y)^2} G_{\mu\nu}(u(x-y))\sigma^{\mu\nu} - u(x^\mu - y^\mu) G_{\mu\nu}(u(x-y))\gamma^\nu \right. \\
& \times \left. \frac{i}{4\pi^2(x-y)^2} - i\frac{m_q}{32\pi^2} G_{\mu\nu}(u(x-y))\sigma^{\mu\nu} \left[\ln \left(-\frac{(x-y)^2 \Lambda^2}{u} + 2\gamma_E \right) \right] \right\}, \quad (14)
\end{aligned}$$

where

$$S_q^{free}(x-y) = \frac{i(\not{x} - \not{y})}{2\pi^2(x-y)^4} - \frac{m_q}{4\pi^2(x-y)^2},$$

is the free quark operator, and $\Lambda = (0.5 - 1.0) \text{ GeV}$ [14] is the scale parameter separating the perturbative and nonperturbative regions. It should be remembered that the light

cone expansion of the light quark propagator is obtained in [13], which gets contributions from nonlocal three $\bar{q}Gq$, and four-particle $\bar{q}q\bar{q}q$, $\bar{q}G^2q$ operators, where $G_{\mu\nu}$ is the gluon strength tensor. Expansion in conformal spin proves that the contributions coming from four-particle operators are small and can be neglected [15].

The expression of the complete heavy quark propagator in the coordinate space is given as,

$$S_Q(x) = S_Q^{free} - \frac{g_s}{16\pi^2} \int_0^1 du G_{\mu\nu}(u(x-y)) \left(i \left[\sigma^{\mu\nu} (\not{x} - \not{y}) + (\not{x} - \not{y}) \sigma^{\mu\nu} \right] \right. \\ \left. \times \frac{K_1(m_Q \sqrt{-(x-y)^2})}{\sqrt{-(x-y)^2}} + 2\sigma^{\mu\nu} K_0(m_Q \sqrt{-(x-y)^2}) \right) \Big\} + \dots, \quad (15)$$

respectively, where $K_i(m_Q \sqrt{-x^2})$ are the modified Bessel functions. The free part of the heavy quark propagator has the following form:

$$S_Q^{free} = \frac{m_Q^2}{4\pi^2} \left\{ \frac{K_1(m_Q \sqrt{-(x-y)^2})}{\sqrt{-(x-y)^2}} + i \frac{(\not{x} - \not{y})}{-(x-y)^2} K_2(m_Q \sqrt{-(x-y)^2}) \right\}.$$

The correlation function contains short distance (perturbative), and long distance (nonperturbative) contributions. The nonperturbative contribution can be obtained by making the replacement,

$$S_{\mu\nu}^{ab}(x-y) \rightarrow -\frac{1}{4} \bar{q}^a(x) \Gamma_\rho q^b(y) (\Gamma_\rho)_{\mu\nu}, \quad (16)$$

in the light quark operator given in Eq. (13), where $\Gamma_\rho = \{1, \gamma_5, \gamma_\mu, i\gamma_5\gamma_\mu, \sigma_{\mu\nu}/\sqrt{2}\}$. Moreover, matrix elements of the nonlocal operators, such as $\bar{q}(x)\Gamma q(y)$, $\bar{q}(x)F_{\mu\nu}\Gamma q(y)$, and $\bar{q}(x)G_{\mu\nu}\Gamma q(y)$ appear between vacuum and photon states, when a photon interacts with the light quark fields at large distance. Parametrization of these matrix elements in terms of the photon distribution amplitudes (DAs) is obtained in [10],

$$\begin{aligned} \langle \gamma(q) | \bar{q}(x) \sigma_{\mu\nu} q(0) | 0 \rangle &= -ie_q \langle \bar{q}q \rangle (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int_0^1 du e^{i\bar{u}qx} \left(\chi \varphi_\gamma(u) + \frac{x^2}{16} \mathbb{A}(u) \right) \\ &\quad - \frac{i}{2(qx)} e_q \langle \bar{q}q \rangle \left[x_\nu \left(\varepsilon_\mu - q_\mu \frac{\varepsilon x}{qx} \right) - x_\mu \left(\varepsilon_\nu - q_\nu \frac{\varepsilon x}{qx} \right) \right] \int_0^1 du e^{i\bar{u}qx} h_\gamma(u) \\ \langle \gamma(q) | \bar{q}(x) \gamma_\mu q(0) | 0 \rangle &= e_q f_{3\gamma} \left(\varepsilon_\mu - q_\mu \frac{\varepsilon x}{qx} \right) \int_0^1 du e^{i\bar{u}qx} \psi^v(u) \\ \langle \gamma(q) | \bar{q}(x) \gamma_\mu \gamma_5 q(0) | 0 \rangle &= -\frac{1}{4} e_q f_{3\gamma} \epsilon_{\mu\nu\alpha\beta} \varepsilon^\nu q^\alpha x^\beta \int_0^1 du e^{i\bar{u}qx} \psi^a(u) \\ \langle \gamma(q) | \bar{q}(x) g_s G_{\mu\nu}(vx) q(0) | 0 \rangle &= -ie_q \langle \bar{q}q \rangle (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{S}(\alpha_i) \\ \langle \gamma(q) | \bar{q}(x) g_s \tilde{G}_{\mu\nu} i\gamma_5(vx) q(0) | 0 \rangle &= -ie_q \langle \bar{q}q \rangle (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \tilde{\mathcal{S}}(\alpha_i) \\ \langle \gamma(q) | \bar{q}(x) g_s \tilde{G}_{\mu\nu}(vx) \gamma_\alpha \gamma_5 q(0) | 0 \rangle &= e_q f_{3\gamma} q_\alpha (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{A}(\alpha_i) \end{aligned}$$

$$\begin{aligned}
\langle \gamma(q) | \bar{q}(x) g_s G_{\mu\nu}(vx) i\gamma_\alpha q(0) | 0 \rangle &= e_q f_{3\gamma} q_\alpha (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{V}(\alpha_i) \\
\langle \gamma(q) | \bar{q}(x) \sigma_{\alpha\beta} g_s G_{\mu\nu}(vx) q(0) | 0 \rangle &= e_q \langle \bar{q}q \rangle \left\{ \left[\left(\varepsilon_\mu - q_\mu \frac{\varepsilon x}{qx} \right) \left(g_{\alpha\nu} - \frac{1}{qx} (q_\alpha x_\nu + q_\nu x_\alpha) \right) q_\beta \right. \right. \\
&\quad - \left(\varepsilon_\mu - q_\mu \frac{\varepsilon x}{qx} \right) \left(g_{\beta\nu} - \frac{1}{qx} (q_\beta x_\nu + q_\nu x_\beta) \right) q_\alpha \\
&\quad - \left(\varepsilon_\nu - q_\nu \frac{\varepsilon x}{qx} \right) \left(g_{\alpha\mu} - \frac{1}{qx} (q_\alpha x_\mu + q_\mu x_\alpha) \right) q_\beta \\
&\quad \left. + \left(\varepsilon_\nu - q_\nu \frac{\varepsilon x}{qx} \right) \left(g_{\beta\mu} - \frac{1}{qx} (q_\beta x_\mu + q_\mu x_\beta) \right) q_\alpha \right] \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{T}_1(\alpha_i) \\
&\quad + \left[\left(\varepsilon_\alpha - q_\alpha \frac{\varepsilon x}{qx} \right) \left(g_{\mu\beta} - \frac{1}{qx} (q_\mu x_\beta + q_\beta x_\mu) \right) q_\nu \right. \\
&\quad - \left(\varepsilon_\alpha - q_\alpha \frac{\varepsilon x}{qx} \right) \left(g_{\nu\beta} - \frac{1}{qx} (q_\nu x_\beta + q_\beta x_\nu) \right) q_\mu \\
&\quad - \left(\varepsilon_\beta - q_\beta \frac{\varepsilon x}{qx} \right) \left(g_{\mu\alpha} - \frac{1}{qx} (q_\mu x_\alpha + q_\alpha x_\mu) \right) q_\nu \\
&\quad \left. + \left(\varepsilon_\beta - q_\beta \frac{\varepsilon x}{qx} \right) \left(g_{\nu\alpha} - \frac{1}{qx} (q_\nu x_\alpha + q_\alpha x_\nu) \right) q_\mu \right] \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{T}_2(\alpha_i) \\
&\quad + \frac{1}{qx} (q_\mu x_\nu - q_\nu x_\mu) (\varepsilon_\alpha q_\beta - \varepsilon_\beta q_\alpha) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{T}_3(\alpha_i) \\
&\quad \left. + \frac{1}{qx} (q_\alpha x_\beta - q_\beta x_\alpha) (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{T}_4(\alpha_i) \right\} \\
\langle \gamma(q) | \bar{q}(x) e_q F_{\mu\nu}(vx) q(0) | 0 \rangle &= -ie_q \langle \bar{q}q \rangle (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{S}^\gamma(\alpha_i) \\
\langle \gamma(q) | \bar{q}(x) \sigma_{\alpha\beta} F_{\mu\nu}(vx) q(0) | 0 \rangle &= e_q \langle \bar{q}q \rangle \frac{1}{qx} (q_\alpha x_\beta - q_\beta x_\alpha) (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \\
&\quad \times \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{T}_4^\gamma(\alpha_i),
\end{aligned}$$

where $\varphi_\gamma(u)$ is the leading twist-2, $\psi^v(u)$, $\psi^a(u)$, \mathcal{A} and \mathcal{V} are the twist-3, and $h_\gamma(u)$, \mathbb{A} , \mathcal{S} , $\tilde{\mathcal{S}}$, \mathcal{S}^γ , \mathcal{T}_i ($i = 1, 2, 3, 4$), \mathcal{T}_4^γ are the twist-4 photon DAs, χ is the magnetic susceptibility, and the measure $\mathcal{D}\alpha_i$ is defined as

$$\int \mathcal{D}\alpha_i = \int_0^1 d\alpha_{\bar{q}} \int_0^1 d\alpha_q \int_0^1 d\alpha_g \delta(1 - \alpha_{\bar{q}} - \alpha_q - \alpha_g).$$

Equating the coefficients of the Lorentz structure $(\varepsilon^\beta q^\nu - \varepsilon^\nu q^\beta)g_{\mu\alpha}$ from both representations of the correlation function, the sum rules for the magnetic moments of the negative parity tensor mesons are obtained. To suppress the contributions of the higher states and continuum, double Borel transformation with respect to the variables $-p^2$ and $-(p+q)^2$ is performed. After this transformation, finally, the magnetic moment of the negative parity tensor mesons is obtained whose explicit expressions are given as,

• Light tensor mesons

$$\begin{aligned}
& \frac{m_T^6 g_T^2}{4} e^{-m_T^2/M^2} G_{M_1}(q^2 = 0) = \\
& \frac{1}{24} M^2 E_0(x) \left[e_{q_1} m_{q_2} \langle \bar{q}_1 q_1 \rangle \left(\mathbb{A}(u_0) + 4u_0 j_1(h_\gamma) - 2\tilde{j}_1(h_\gamma) \right) \right. \\
& \left. - e_{q_2} m_{q_1} \langle \bar{q}_2 q_2 \rangle \left(\mathbb{A}(\bar{u}_0) + 2j_2(h_\gamma) + \tilde{j}_2(h_\gamma) \right) \right] \\
& - \frac{1}{48\pi^2} M^4 E_1(x) (3 - 4u_0) (e_{q_1} - e_{q_2}) m_{q_1} m_{q_2} \\
& - \frac{1}{48} f_{3\gamma} M^4 E_1(x) \left[e_{q_2} \left(8j_2(\psi_v) - \psi_a(\bar{u}_0) + 4\psi_v(\bar{u}_0) + \psi'_a(\bar{u}_0) \right) \right. \\
& \left. - e_{q_1} \left(8j_1(\psi_v) - \psi_a(u_0) + 2u_0(4\psi_v(u_0) - \psi'_a(u_0)) \right) \right] \\
& - \frac{1}{240\pi^2} M^6 E_2(x) (5 - 18u_0) (e_{q_1} - e_{q_2}) \\
& - \frac{1}{72} m_0^2 [e_{q_2} \langle \bar{q}_1 q_1 \rangle (m_{q_1} - 3m_{q_2}) + e_{q_1} \langle \bar{q}_2 q_2 \rangle (3m_{q_1} - m_{q_2})] . \tag{17}
\end{aligned}$$

• **Heavy tensor mesons**

$$\begin{aligned}
& \frac{m_{T_Q}^6 g_{T_Q}^2}{4} e^{-m_{T_Q}^2/M^2} G_{M_1}(q^2 = 0) = \\
& \frac{1}{1152\pi^2} \left[e_q \langle g_s^2 G^2 \rangle M^2 (2m_Q^2 \mathcal{I}_2 - m_Q^4 \mathcal{I}_3) \right] - \frac{e^{-m_Q^2/M^2}}{3456m_Q\pi^2} M^2 \left\{ 9m_Q (e_q \langle g_s^2 G^2 \rangle - 96e_Q m_Q \pi^2 \langle \bar{q}q \rangle) \right. \\
& - 4e_q \pi^2 \left[18m_Q^2 \langle \bar{q}q \rangle \left(\mathbb{A}(u_0) + 2\tilde{j}_1(h_\gamma) + 4\tilde{j}_2(h_\gamma) \right) + \langle g_s^2 G^2 \rangle \langle \bar{q}q \rangle \chi\varphi_\gamma(u_0) - 36f_{3\gamma} m_Q^3 \psi^a(u_0) \right] \Big\} \\
& + \frac{1}{32\pi^2} e_Q m_Q^4 M^4 (\mathcal{I}_2 - m_Q^2 \mathcal{I}_3) + \frac{e^{-m_Q^2/M^2}}{96} e_q M^4 \left\{ 8f_{3\gamma} \tilde{j}_1(\psi^v) - 8m_Q \langle \bar{q}q \rangle \chi\varphi_\gamma(u_0) \right. \\
& - f_{3\gamma} [6\psi^a(u_0) - 4\psi^v(u_0) + \psi^{a'}(u_0)] \Big\} + \frac{e^{-m_Q^2/M^2}}{16\pi^2} e_q M^6 \\
& + \frac{1}{32\pi^2} m_Q^2 M^6 \left[2e_Q \mathcal{I}_2 - 3e_Q m_Q^2 \mathcal{I}_3 - 4e_Q m_Q^4 \mathcal{I}_4 - 2e_q m_Q^4 \mathcal{I}_4 - 2(e_Q - e_q) m_Q^6 \mathcal{I}_5 \right] \\
& - \frac{e^{-m_Q^2/M^2}}{6912M^2} m_Q \left\{ 432e_Q m_0^2 m_Q^2 \langle \bar{q}q \rangle - e_q \langle g_s^2 G^2 \rangle \left[4\langle \bar{q}q \rangle \mathbb{A}(u_0) - 4(5 - 4u_0) \langle \bar{q}q \rangle \tilde{j}_1(h_\gamma) \right. \right. \\
& - 40\langle \bar{q}q \rangle \tilde{j}_2(h_\gamma) + m_Q \left(8m_Q \langle \bar{q}q \rangle \chi\varphi_\gamma(u_0) - f_{3\gamma} (8\tilde{j}_1(\psi^v) - 6\psi^a(u_0) - 4\psi^v(u_0) - \psi^{a'}(u_0)) \right) \Big] \Big\} \\
& - \frac{e^{-m_Q^2/M^2}}{3456M^4} e_q \langle g_s^2 G^2 \rangle m_Q^3 \left[\langle \bar{q}q \rangle \left(\mathbb{A}(u_0) + 2\tilde{j}_1(h_\gamma) + 4\tilde{j}_2(h_\gamma) \right) - 2f_{3\gamma} m_Q \psi^a(u_0) \right] \\
& - \frac{e^{-m_Q^2/M^2}}{3456M^6} e_q \langle g_s^2 G^2 \rangle m_Q^5 \langle \bar{q}q \rangle \mathbb{A}(u_0) - \frac{e^{-m_Q^2/M^2}}{3456m_Q\pi^2} e_q \left\{ 4(\langle g_s^2 G^2 \rangle - 18m_Q^4) \pi^2 \langle \bar{q}q \rangle \mathbb{A}(u_0) \right. \\
& + \langle g_s^2 G^2 \rangle \left[3m_Q^3 + \pi^2 \left(4(2 + u_0) \langle \bar{q}q \rangle \tilde{j}_1(h_\gamma) + 16\langle \bar{q}q \rangle \tilde{j}_2(h_\gamma) - m_Q \{ 12m_Q \langle \bar{q}q \rangle \chi\varphi_\gamma(u_0) \right. \right. \\
& \left. \left. - f_{3\gamma} [12\tilde{j}_1(\psi^v) + 2\psi^a(u_0) + (2 - u_0)(4\psi^v(u_0) - \psi^{a'}(u_0))] \right) \right] \Big\} , \tag{18}
\end{aligned}$$

where

$$u_0 = \frac{M_1^2}{M_1^2 + M_2^2} , \quad M^2 = \frac{M_1^2 M_2^2}{M_1^2 + M_2^2} .$$

The functions $j_n(f(u))$, and $\tilde{j}_1(f(u))$ ($n = 1, 2$) are defined as:

$$\begin{aligned}
j_1(f(u')) &= \int_{u_0}^1 du' f(u') , \\
\tilde{j}_1(f(u')) &= \int_{u_0}^1 du' (u' - u_0) f(u') , \\
j_2(f(u')) &= \int_0^{\bar{u}_0} du' f(u') , \\
\tilde{j}_2(f(u')) &= \int_0^{\bar{u}_0} du' (u' - \bar{u}_0) f(u') , \\
E_n(x) &= 1 - e^{-x} \sum_{k=0}^n \frac{x^k}{k!} = \frac{1}{n!} \int_0^x dx' x'^n e^{-x'} , \\
\mathcal{I}_n &= \int_{m_Q^2}^{s_0} ds \frac{e^{-s/M^2}}{s^n} ,
\end{aligned}$$

with $x = s_0/M^2$, s_0 being the continuum threshold, and the Borel parameter M^2 is defined as,

$$M^2 = \frac{M_1^2 M_2^2}{M_1^2 + M_2^2} \text{ and, } u_0 = \frac{M_1^2}{M_1^2 + M_2^2} .$$

Since we have the same heavy tensor mesons in the initial and final states, we can set $M_1^2 = M_2^2 = 2M^2$, as the result of which we have,

$$u_0 = \frac{M_1^2}{(M_1^2 + M_2^2)} = \frac{1}{2} .$$

3 Numerical analysis

In this section we perform the numerical analysis of the sum rules for the magnetic dipole moments of the negative parity tensor mesons derived in the previous section. The input parameters used in the numerical analysis are, $\langle \bar{u}u \rangle(\mu = 1 \text{ GeV}) = \langle \bar{d}d \rangle(\mu = 1 \text{ GeV}) = -(0.243)^3 \text{ GeV}^3$, $\langle \bar{s}s \rangle|_{\mu=1 \text{ GeV}} = (0.8 \pm 0.2) \langle \bar{u}u \rangle(\mu = 1 \text{ GeV})$, $m_0^2 = (0.8 \pm 0.2) \text{ GeV}^2$ which are obtained from the mass sum rule analysis for the light baryons [16, 17], and B meson [18]. Furthermore, we have used the \overline{MS} values of the heavy quarks masses whose values are $\bar{m}_b(\bar{m}_b) = (4.16 \pm 0.03) \text{ GeV}$ and $\bar{m}_c(\bar{m}_c) = (1.28 \pm 0.03) \text{ GeV}$ [19, 20]. The magnetic susceptibility of quarks is calculated in framework of the QCD sum rules in [21–23]. As we have already noted, the masses of the negative parity tensor mesons are calculated in [6], which we have used in the present work. We further have calculated the decay constants of $J = 2^-$ mesons which are needed in the numerical analysis.

The key input parameters in the present numerical analysis are the DAs. Below we present only the expressions of the DAs that enter to the sum rules for the magnetic dipole moments.

$$\varphi_\gamma(u) = 6u\bar{u} \left[1 + \varphi_2(\mu) C_2^{\frac{3}{2}}(u - \bar{u}) \right] ,$$

$$\begin{aligned}
\psi^v(u) &= 3[3(2u-1)^2 - 1] + \frac{3}{64}(15w_\gamma^V - 5w_\gamma^A)[3 - 30(2u-1)^2 + 35(2u-1)^4] , \\
\psi^a(u) &= [1 - (2u-1)^2][5(2u-1)^2 - 1]\frac{5}{2}\left(1 + \frac{9}{16}w_\gamma^V - \frac{3}{16}w_\gamma^A\right) , \\
\mathcal{A}(\alpha_i) &= 360\alpha_q\alpha_{\bar{q}}\alpha_g^2\left[1 + w_\gamma^A\frac{1}{2}(7\alpha_g - 3)\right] , \\
\mathcal{V}(\alpha_i) &= 540w_\gamma^V(\alpha_q - \alpha_{\bar{q}})\alpha_q\alpha_{\bar{q}}\alpha_g^2 , \\
h_\gamma(u) &= -10(1 + 2\kappa^+)C_2^{\frac{1}{2}}(u - \bar{u}) , \\
\mathbb{A}(u) &= 40u^2\bar{u}^2(3\kappa - \kappa^+ + 1) + 8(\zeta_2^+ - 3\zeta_2)[u\bar{u}(2 + 13u\bar{u}) + 2u^3(10 - 15u + 6u^2)\ln(u) \\
&\quad + 2\bar{u}^3(10 - 15\bar{u} + 6\bar{u}^2)\ln(\bar{u})] . \tag{19}
\end{aligned}$$

The values of the constant parameters in the DAs are given as: $\varphi_2(1 \text{ GeV}) = 0$, $w_\gamma^V = 3.8 \pm 1.8$, $w_\gamma^A = -2.1 \pm 1.0$, $\kappa = 0.2$, $\kappa^+ = 0$, $\zeta_1 = 0.4$, $\zeta_2 = 0.3$, $\zeta_1^+ = 0$ and $\zeta_2^+ = 0$ [10].

The sum rules for the magnetic dipole moments of the negative parity $J^{PC} = 2^{--}$ tensor mesons which are obtained in the previous section contain two more arbitrary parameters, in addition to the input parameters summarized above: Borel mass parameter M^2 and the continuum threshold s_0 . In the analysis of the sum rules, the working regions of these two parameters should be determined, such that the magnetic dipole moments exhibit weak dependence on these parameters. The working regions should satisfy the following requirements: The upper limit of M^2 is determined from the condition that the higher states contributions constitute maximum 40% of the perturbative ones. The lower bound of M^2 is obtained by requiring that the OPE should be convergent, i.e., the higher twist contributions should be less than the leading twist contributions. From these conditions we have obtained the working regions of the $J^{PC} = 2^{--}$ tensor mesons, which is listed in Table 1.

	$M^2 \text{ (GeV}^2\text{)}$	$s_0 \text{ (GeV}^2\text{)}$
$\bar{q}q$	1.3÷1.8	2.1 ²
$\bar{q}s$	1.4÷2.0	2.2 ²
$\bar{s}s$	1.5÷2.2	2.4 ²
$\bar{q}c$	2.0÷4.0	3.3 ²
$\bar{s}c$	2.2÷4.2	3.6 ²
$\bar{q}b$	4.5÷7.0	6.2 ²
$\bar{s}b$	4.7÷8.0	7.0 ²

Table 1: The working regions of the Borel parameter M^2 , and the corresponding values of the continuum threshold s_0 for the $J^{PC} = 2^{--}$ tensor mesons (these values are taken from [6])

The values of the continuum threshold listed in Table 1, are determined in [6] from the analysis of the two-point correlation function.

Using the values of the input parameters and the working regions of M^2 and s_0 , the values of the magnetic dipole moments can be determined. As an example, in Figs. (1) and (2) we present the dependence of the magnetic dipole moments of K_2^+ and \mathcal{D}_2^0 mesons on M^2 at several fixed values of s_0 , respectively. It follows from these figures that the magnetic dipole moments show weak dependence on M^2 in its working region. Similar analysis for the other $J^{PC} = 2^{--}$ tensor mesons are carried out whose results are presented in Table 2.

For completeness, we also present the values of the magnetic dipole moments for the positive parity tensor mesons in the same table. From the comparison of the results we deduce that:

- In the case of light tensor mesons, the magnetic dipole moments of the negative parity mesons are 2–5 times smaller compared to that for the positive parity mesons.
- For the heavy tensor mesons, however, the situation is to the contrary namely, the magnetic moments of the negative parity tensor mesons are larger compared to the ones for the positive parity mesons.

These results can be explained by the fact that, the terms proportional to the quark mass in the expressions of the sum rules have opposite sign. Therefore, the contributions coming from the heavy quark mass terms are constructive (destructive) for the negative (positive) parity tensor mesons. Additionally, this difference can be attributed to the differences in masses and residues of the tensor mesons of both parities.

In summary, the magnetic dipole moments of the light and heavy $J^{PC} = 2^{--}$ tensor mesons are calculated in framework of the light cone QCD sum rules method. Comparison of the predictions for the magnetic dipole moments of the negative and positive parity mesons is also presented. It is observed that, the results for the magnetic dipole moments of the negative parity light mesons are smaller compared to the ones for the corresponding positive parity tensor mesons, while the situation is to the contrary for the heavy tensor mesons.

	$G_{M_1}(\mu_N)$	$G_M(\mu_N)$
Tensor mesons	Negative parity	Positive parity
f_2^\pm	0	0
a_2^+	0.26 ± 0.05	1.28 ± 0.27
a_2^0	0	0
K_2^+	0.26 ± 0.05	0.5 ± 0.07
K_2^0	-0.015 ± 0.003	0.05 ± 0.007
\mathcal{D}_2^0	1.97 ± 0.16	0.3 ± 0.1
\mathcal{D}_2^+	1.64 ± 0.33	-0.80 ± 0.08
$\mathcal{D}_{2_s}^+$	2.5 ± 0.6	-0.80 ± 0.73
\mathcal{B}_2^+	1.33 ± 0.50	0.62 ± 0.11
\mathcal{B}_2^0	-1.50 ± 0.33	-0.20 ± 0.05
$\mathcal{B}_{2_s}^+$	-0.66 ± 0.22	-0.23 ± 0.05

Table 2: The values of the magnetic dipole moments of the negative and positive parity tensor mesons in units of the nuclear magneton μ_N .

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Figure captions

Fig. (1) Dependence of the magnetic dipole moment of the negative parity K_2^+ tensor meson, on Borel mass square M^2 , at several fixed values of the continuum threshold, in units of $(e/2m_T)$.

Fig. (2) The same as Fig. (1), but for the negative parity \mathcal{D}_2^0 tensor meson.

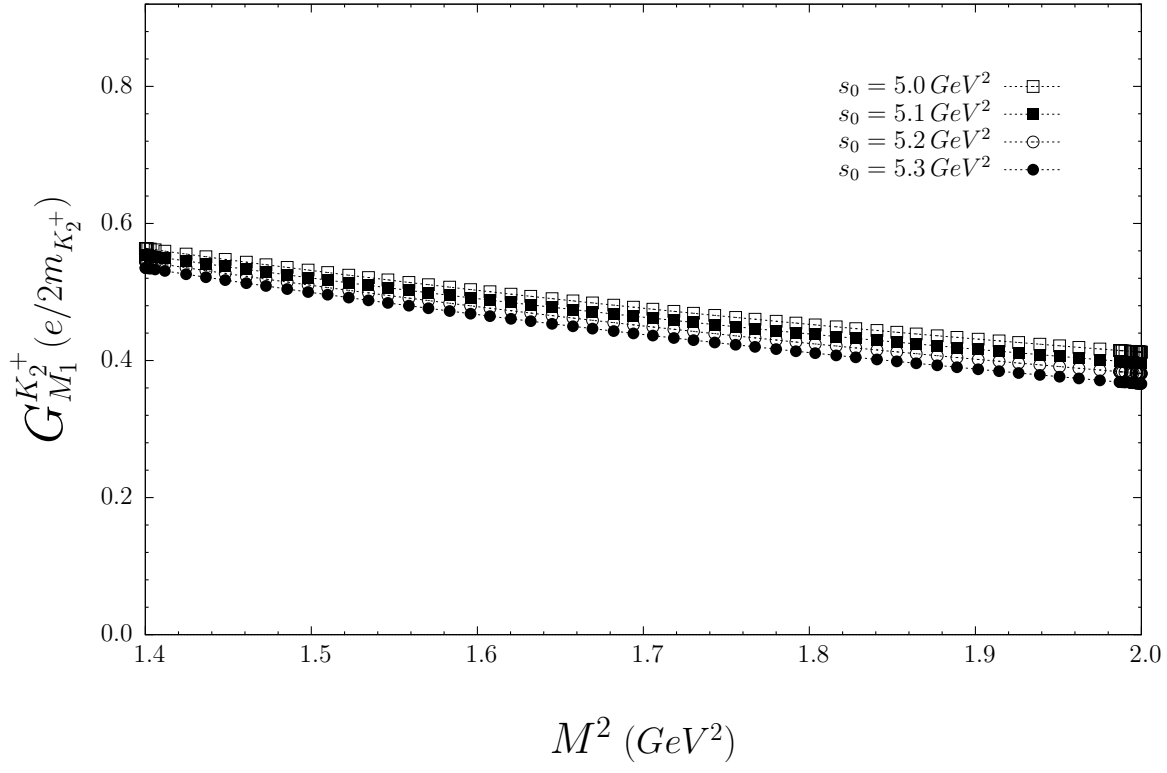


Figure 1:

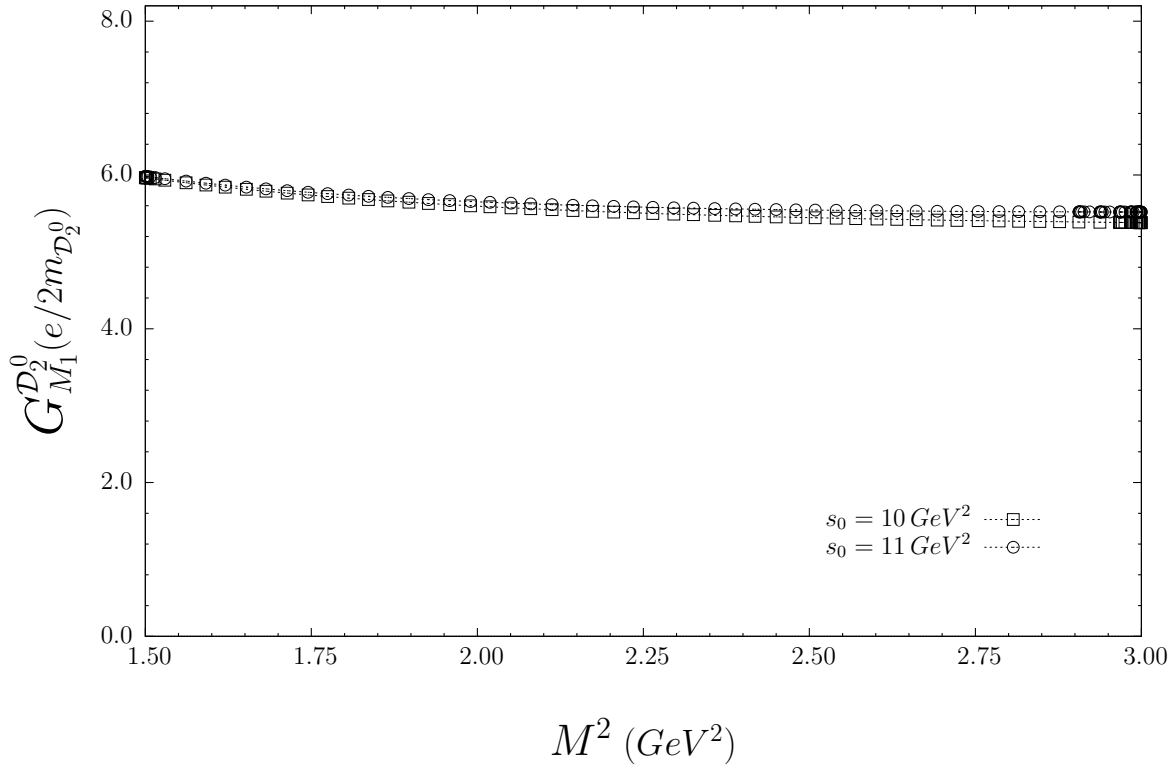


Figure 2: